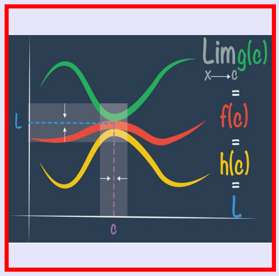
Calculus I

Lecture 14

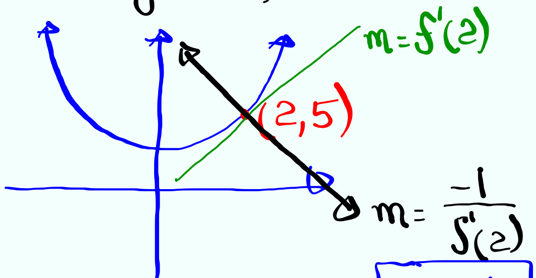


Feb 19-8:47 AM

Class Quiz 14

Find the equation of the $\boxed{\text{normal line}}$ on the Curve given by $f(x) = x^2 + 1$ at $\boxed{x = 2}$

Answer in Slope-Int. Form



$\boxed{m = \frac{-1}{4}}$

$f(x) = x^2 + 1$

$f'(x) = 2x$

$f'(2) = 4$

$y - y_1 = m(x - x_1)$

$y - 5 = \frac{-1}{4}(x - 2)$

$y - 5 = \frac{-1}{4}x + \frac{1}{2}$

$\boxed{y = \frac{-1}{4}x + \frac{11}{2}}$

Apr 2-8:01 AM

Given $f(3)=5$, $f'(3)=-4$

find $g'(3)$ if $g(x)=x^2\cdot f(x)$

$$g'(x)=2x\,f(x)+x^2 f'(x)$$

$$g'(3)=2\cdot3\cdot f(3)+3^2\cdot f'(3)$$

$$=6\cdot5+9\cdot(-4)$$

$$=30-36=\boxed{-6}$$

Find $h'(3)$ if $h(x)=\dfrac{x^2-2x}{f(x)}$

$$h'(x)=\frac{(2x-2)\cdot f(x)-(x^2-2x)\cdot f'(x)}{[f(x)]^2}$$

$$h'(3)=\frac{(2\cdot3-2)\cdot f(3)-(3^2-2\cdot3)\cdot f'(3)}{[f(3)]^2}=\frac{4\cdot5-3\cdot(-4)}{(5)^2}$$

$$=\frac{20+12}{25}=\boxed{\frac{32}{25}}$$

Apr 2-9:02 AM

Find $f'(x)$

1) $f(x)=(x^3-4x^2+8)^4$

$$f'(x)=4(x^3-4x^2+8)^3\cdot(3x^2-8x)$$

2) $f(x)=\sqrt{\dfrac{x-1}{x+1}}$ $\rightarrow$ $f(x)=\left(\dfrac{x-1}{x+1}\right)^{1/2}$

$$f'(x)=\frac{1}{2}\left(\frac{x-1}{x+1}\right)^{-1/2}\cdot\frac{1(x+1)-(x-1)\cdot1}{(x+1)^2}$$

$$f'(x)=\frac{1}{2}\cdot\left(\frac{x+1}{x-1}\right)^{\frac{1}{2}}\cdot\frac{\cancel{x}+1-\cancel{x}+1}{(x+1)^2}$$

$$=\frac{1}{\cancel{2}}\cdot\frac{(x+1)^{1/2}}{(x-1)^{1/2}}\cdot\frac{\cancel{2}\,1}{(x+1)^2}$$

$$=\frac{1}{(x-1)^{1/2}(x+1)^{2-1/2}}$$

$$=\frac{1}{(x-1)^{1/2}(x+1)^{3/2}}$$

Apr 2-9:10 AM

find $f'(x)$

1) $f(x) = (\sin x + \cos x)^2$

$f'(x) = 2(\sin x + \cos x)^{1} \cdot (\cos x - \sin x)$

$= 2(\overset{A}{\cos x} \overset{+B}{+ \sin x})(\overset{A}{\cos x} \overset{-B}{- \sin x})$

$= 2[\overset{A^2 - B^2}{\cos^2 x - \sin^2 x}]$

$= \boxed{2\cos 2x}$

2) $f(x) = \sin^2 x + \cos^2 x$

$f(x) = 1$ $\boxed{f'(x) = 0}$

3) $f(x) = \sin x^2 + \cos x^2$

$f'(x) = \cos x^2 \cdot 2x + -\sin x^2 \cdot 2x$

$f'(x) = 2x(\cos x^2 - \sin x^2)$

Apr 2-9:21 AM

find $f''(x)$ $f''(x) = \frac{d}{dx}[f'(x)]$

1) $f(x) = x^3$ $f'(x) = 3x^2$ $f''(x) = 3 \cdot 2x = \boxed{6x}$

$\sqrt[n]{x} = x^{\frac{1}{n}}$

2) $f(x) = \sqrt[3]{x}$ $f(x) = x^{1/3}$

$f'(x) = \frac{1}{3}x^{\frac{1}{3}-1}$ $f'(x) = \frac{1}{3}x^{-2/3}$

$f''(x) = \frac{1}{3} \cdot \frac{-2}{3} \cdot x^{-2/3-1} = \frac{-2}{9}x^{-5/3}$

$f''(x) = \frac{-2}{9x^{5/3}}$

$= \frac{-2}{9\sqrt[3]{x^5}} = \frac{-2}{9\sqrt[3]{x^3}\sqrt[3]{x^2}}$

$= \boxed{\frac{-2}{9x\sqrt[3]{x^2}}}$

Apr 2-9:33 AM

Find $f'(x)$ Do not simplify

1) $f(x) = \sin(\tan(x^3))$

$f'(x) = \cos(\tan x^3) \cdot \sec^2 x^3 \cdot \boxed{3x^2}$

$$\boxed{f'(x) = 3x^2 \cos(\tan x^3) \cdot \sec^2 x^3}$$

2) $f(x) = \tan(\sin\sqrt{x}) - \sec(\cos\sqrt{x})$

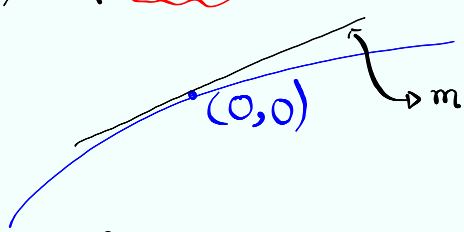
$f'(x) = \sec^2(\sin\sqrt{x}) \cdot \cos\sqrt{x} \cdot \boxed{\frac{1}{2\sqrt{x}}} - \sec(\cos\sqrt{x}) \cdot \tan(\cos\sqrt{x}) \cdot -\sin\sqrt{x} \cdot \boxed{\frac{1}{2\sqrt{x}}}$

$= \frac{1}{2\sqrt{x}}\left[\sec^2(\sin\sqrt{x}) \cdot \cos\sqrt{x} + \sec(\cos\sqrt{x}) \cdot \tan(\cos\sqrt{x}) \cdot \sin\sqrt{x}\right]$

Apr 2-9:41 AM

Find eqn of tan. line to the graph of

$y = \sin x + \sin^2 x$ at $x = 0$. $(\sin x)^2$



$m = \left.\frac{dy}{dx}\right|_{(0,0)}$

$= \boxed{1}$

$y' = \cos x + 2\sin x \cdot \cos x$

$y' = \cos x + \sin 2x$

$y - y_1 = m(x - x_1)$

$y - 0 = 1(x - 0)$

$\boxed{y = x}$

Apr 2-9:53 AM

$$\frac{d}{dx}\left[f(x)+g(x)\right]=\frac{d}{dx}\left[f(x)\right]+\frac{d}{dx}\left[g(x)\right]$$

$$\frac{d}{dx}\left[f(x)-g(x)\right]=\frac{d}{dx}\left[f(x)\right]-\frac{d}{dx}\left[g(x)\right]$$

$$\frac{d}{dx}\left[f(x)\cdot g(x)\right]=f'(x)\cdot g(x)+f(x)\cdot g'(x)$$

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]=\frac{f'(x)\cdot g(x)-f(x)\cdot g'(x)}{\left[g(x)\right]^2}$$

Apr 2-10:02 AM

Prove $\frac{d}{dx}\left[f(x)\cdot g(x)\right]=f'(x)\cdot g(x)+f(x)\cdot g'(x)$

Let $F(x)=f(x)\cdot g(x)$

$$F'(x)=\lim_{h\to 0}\frac{F(x+h)-F(x)}{h}$$

$$=\lim_{h\to 0}\frac{f(x+h)g(x+h)-f(x)g(x)}{h}$$

$$=\lim_{h\to 0}\frac{f(x+h)g(x+h)-f(x)g(x+h)+f(x)g(x+h)-f(x)g(x)}{h}$$

$$=\lim_{h\to 0}\frac{g(x+h)\cdot\left[f(x+h)-f(x)\right]+f(x)\left[g(x+h)-g(x)\right]}{h}$$

$$=\lim_{h\to 0}\left[\frac{g(x+h)\cdot\left[f(x+h)-f(x)\right]}{h}+\frac{f(x)\cdot\left[g(x+h)-g(x)\right]}{h}\right]$$

$$=\lim_{h\to 0}\frac{g(x+\cancel{h})\cdot\left[f(x+h)-f(x)\right]}{h}+\lim_{h\to 0}\frac{f(x)\cdot\left[g(x+h)-g(x)\right]}{h}$$

$$=\boxed{f'(x)\cdot g(x)+f(x)\cdot g'(x)}$$

Apr 2-10:05 AM

Implicit Differentiation

$y = \sqrt{x}$ $\qquad \frac{dy}{dx} = \frac{1}{2\sqrt{x}}$

$y^2 = x$

$\frac{d}{dx}[y^2] = \frac{d}{dx}[x]$

$2y \cdot \frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{2y}$

$$\boxed{\frac{dy}{dx} = \frac{1}{2\sqrt{x}}}$$

Apr 2-10:15 AM

$x^2 + y^2 = 25$

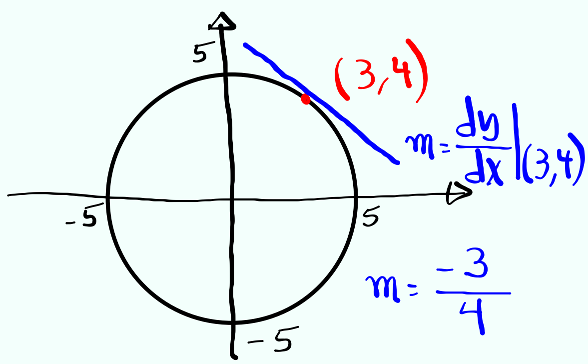
$\frac{d}{dx}[x^2 + y^2] = \frac{d}{dx}[25]$

$\frac{d}{dx}[x^2] + \frac{d}{dx}[y^2] = 0$

$2x + 2y \cdot y' = 0$

$y' = \frac{-2x}{2y}$

$y' = \frac{-x}{y}$



$m = \frac{-3}{4}$

$y - 4 = \frac{-3}{4}(x-3)$

$y = \frac{-3}{4}x + \frac{9}{4} + 4$

$$\boxed{y = \frac{-3}{4}x + \frac{25}{4}}$$

Apr 2-10:20 AM

Given $\sqrt{x} - \sqrt{y} = 2$

Find $\frac{dy}{dx}$.

$$x^{1/2} - y^{1/2} = 2$$

$$\frac{d}{dx}\left[x^{1/2} - y^{1/2}\right] = \frac{d}{dx}[2]$$

$$\frac{1}{2}x^{-1/2} - \frac{1}{2}y^{-1/2}\cdot\frac{dy}{dx} = 0$$

Multiply by 2.

$$x^{-1/2} - y^{-1/2}\frac{dy}{dx} = 0$$

$$-y^{-1/2}\frac{dy}{dx} = -x^{-1/2}$$

$$\frac{dy}{dx} = \frac{\cancel{-}x^{-1/2}}{\cancel{-}y^{-1/2}} = \frac{y^{1/2}}{x^{1/2}}$$

$$\frac{dy}{dx} = \frac{\sqrt{y}}{\sqrt{x}}$$

Apr 2-10:26 AM

$$x^2 + xy - y^2 = 4$$

Find $\frac{dy}{dx}$

$$\frac{d}{dx}\left[x^2 + \boxed{xy} - y^2\right] = \frac{d}{dx}[4]$$

$$2x + 1\cdot y + x\cdot\frac{dy}{dx} - 2y\cdot\frac{dy}{dx} = 0$$

$$x\frac{dy}{dx} - 2y\frac{dy}{dx} = -2x - y$$

$$(x - 2y)\frac{dy}{dx} = -2x - y$$

$$\frac{dy}{dx} = \frac{-2x - y}{x - 2y}$$

Evaluate $\left.\frac{dy}{dx}\right|_{(2,0)}$

$$\left.\frac{dy}{dx}\right|_{(2,0)} = \frac{-2(2) - 0}{2 - 2\cdot 0} = \frac{-4}{2} = \boxed{-2}$$

Apr 2-10:33 AM

Find $\frac{dy}{dx}$ if $\cos x = \sin y$

$$\frac{d}{dx}\left[\cos x\right] = \frac{d}{dx}\left[\sin y\right]$$

$$-\sin x = \cos y \cdot \frac{dy}{dx}$$

$$\boxed{\frac{dy}{dx} = -\frac{\sin x}{\cos y}}$$

Apr 2-10:40 AM

$y = cx^2$ $\qquad$ $x^2 + 2y^2 = K$

1) Find $\frac{dy}{dx}$ for both.

$y = cx^2$

$\frac{dy}{dx} = 2cx$

$x^2 + 2y^2 = K$

$2x + 4y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-2x}{4y}$ $\qquad$ $\frac{dy}{dx} = \frac{-x}{2y}$

2) Multiply these 2 $\frac{dy}{dx}$, and Simplify.

$$\cancel{2}cx \cdot \frac{-x}{\cancel{2}y} = \frac{-cx^2}{y} = \frac{-y}{y} = \boxed{-1}$$

Apr 2-10:44 AM

$y = ax^3$ $\qquad$ $x^2 + 3y^2 = b$

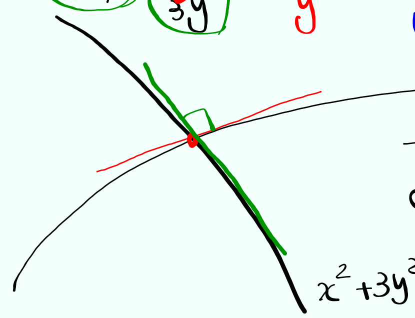
1) Find $\frac{dy}{dx}$ for each.

$\frac{dy}{dx} = 3ax^2$ $\qquad$ $2x + 6y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-x}{3y}$

2) Multiply your findings & simplify.

$3ax^2 \cdot \frac{-x}{3y} = \frac{-ax^3}{y} = -\frac{y}{y} = -1$



$y = ax^3$

these two curves are orthogonal curves.

$x^2 + 3y^2 = b$

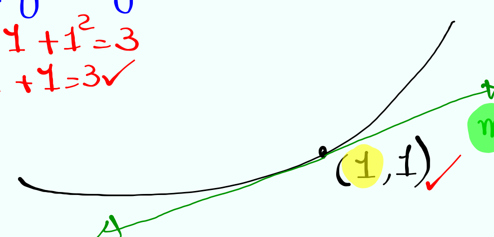
Apr 2-10:51 AM

Find eqn of the tan. line to the graph of

$x^2 + xy + y^2 = 3$ at $(1,1)$.

$1^2 + 1 \cdot 1 + 1^2 = 3$

$1 + 1 + 1 = 3$ ✓



$(1,1)$ ✓

$m = \left.\frac{dy}{dx}\right|_{(1,1)}$

$x^2 + \boxed{xy} + y^2 = 3$

$2x + 1 \cdot y + x \cdot \frac{dy}{dx} + 2y\frac{dy}{dx} = 0$

$2(1) + 1 \cdot 1 + 1 \cdot m + 2 \cdot 1 \cdot m = 0$

$3 + 3m = 0$

$\boxed{m = -1}$

$y - y_1 = m(x - x_1)$

$y - 1 = -1(x - 1)$

$\boxed{y = -x + 2}$

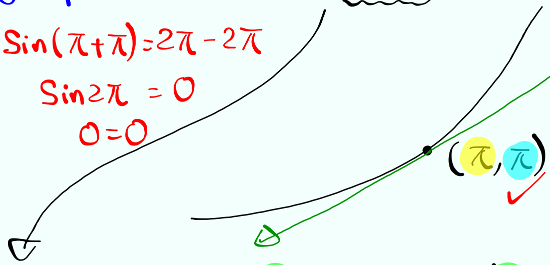
Apr 2-11:01 AM

Find eqn of tan. line at (π, π) to the graph of $\sin(x+y) = 2x - 2y$.

$\sin(\pi+\pi) = 2\pi - 2\pi$

$\sin 2\pi = 0$

$0 = 0$



$m = \left.\frac{dy}{dx}\right|_{(\pi,\pi)}$

$$\cos(x+y)\cdot\left[1+\frac{dy}{dx}\right] = 2 - 2\frac{dy}{dx}$$

$$\underbrace{\cos(\pi+\pi)}_{\cos 2\pi \,=\, 1}\cdot\left[1+m\right] = 2 - 2m$$

$$1\cdot(1+m) = 2-2m$$

$$1+m = 2-2m$$

$$3m = 1 \rightarrow m = \frac{1}{3}$$

$$y - \pi = \frac{1}{3}(x-\pi)$$

$$y = \frac{1}{3}x - \frac{\pi}{3} + \pi$$

$$\boxed{y = \frac{1}{3}x + \frac{2\pi}{3}}$$

Apr 2-11:08 AM